ever, that if radiation protection ultimately cannot be assured within the design limits of the satellite, the options which are left necessitate a reiteration of the whole or part of the procedure. It should also be emphasized that the survival time prediction techniques based on equipment irradiation and sector analysis are not precise. If the survival time is of the same magnitude as the mission life, then redesign is necessary for radiation protection assurance. At the present time an estimated survival time of 2–3 times longer than the design mission life is preferred. It is also possible that the use of a less radiation sensitive transistor with a poorer microwave performance is a preferable solution to the use of high-performance devices which may degrade more quickly.

V. Conclusions

It has been shown that if the ionizing radiation environment is not taken into consideration during the design of MIC linear transistor amplifiers for space applications, the survival time of the equipment may be considerably less than the mission life. One means of overcoming this problem is the use of dedicated active bias networks for each stage of the amplifier. This approach requires more components and increased box dimensions (with a weight penalty) and may lead to a reduction in reliability. Most bias networks presently used are passive, and, if this design approach is used in future applications, a more optimum procedure similar to that outlined in Fig. 14 is required in order to ensure radiation hardness. Although the practical examples given refer to the geosynchronous orbit, the same design approach is applicable to other missions where the appropriate radiation environment can be estimated.

Acknowledgment

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Sensitivity Analysis of Coupled Microstrip Directional Couplers

SRIGIRIPURAM D. SHAMASUNDARA, STUDENT MEMBER, IEEE, AND K. C. GUPTA, SENIOR MEMBER, IEEE

Abstract—Sensitivities of the parameters (coupling, bandwidth, and impedance) of a coupled-line directional coupler with respect to even- and odd-mode impedances have been evaluated. These are used to determine the accuracy required from the closed-form expressions for even- and odd-mode impedances of microstriplines. Closed-form expressions satisfactory for a coupling coefficient greater than 0.3 are proposed and used for evaluating the effect of dimensional tolerances on the performance of microstrip directional couplers. This effect is significant when compared with effects of unequal phase velocities and dispersion.

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The authors are with the Department of Electrical Engineering, I.I.T. Kanpur, Kanpur 208016 India.

I. Introduction

COUPLED microstriplines are used extensively in microwave and millimeter-wave circuits [1] to design directional couplers, filters, impedance transformers, and delay lines. There have been many calculations available for even- and odd-mode impedances of coupled lines [2]–[7]. The earliest and presumably the most widely used analysis (neglecting dispersion) is by Bryant and Weiss [2]. This method requires extensive computations [7], and the results have been tabulated for some specific values of the dielectric constant ($\epsilon_r = 1, 6, 9, 12, 16, 30, 80$) [7], [8]. Schwarzmann [3] has proposed closed-form relations for the even- and the odd-mode impedances. These expressions have limited use in practice because the accuracy of these formulas when compared with the results of Bryant and Weiss [8] is about 20 percent.

Closed-form expressions for coupled-line parameters are required for the sensitivity analysis of microwave circuit components [9] and are useful in computer-aided analysis and design of microwave circuits. Thus there is a need for accurate closed-form expressions.

In this paper, sensitivity analysis of a coupled microstripline directional coupler is carried out in terms of even- and odd-mode impedances. The accuracy desired from the impedance formulas for a given maximum fractional change in coupler performance is determined. New closed-form formulas for even- and odd-mode impedances which give the desired accuracy are proposed. These formulas have an accuracy of 2 percent for even-mode and 2.5 percent for odd-mode impedances when compared with the results of Bryant and Weiss [8] over a wide range of permittivity of the substrate ($\epsilon_r = 1$–80) and in the practical ranges of line width and spacing ($W/h = 0.2$–2.0 and $S/h = 0.05$–2.0).

The closed-form formulas derived here are used in the sensitivity analysis of coupled microstrip directional couplers. Sensitivities of even- and odd-mode impedances with respect to various parameters (ϵ_r, h, W, S) have been calculated. The effect of tolerances on coupler parameters is compared with the effects of unequal phase velocities and dispersion.

II. Sensitivity Analysis of a Coupler

The sensitivity of a parameter A with respect to another parameter B is defined as [10]

$$S_B^A = \lim_{\Delta B \to 0} \frac{\Delta A/A}{\Delta B/B} = \frac{B}{A}\frac{\partial A}{\partial B}. \tag{1}$$

The sensitivity may be utilized to determine the change in circuit characteristics for a given tolerance in a parameter. It is informative to study the variations in the characteristics of a coupler for specified variations in Z_{0e} and Z_{0o}, the even- and odd-mode impedances, respectively. These variations may be due to inaccuracies in calculations for Z_{0e} and Z_{0o} or due to fabrication tolerances or measurement errors. This analysis will yield the accuracy required in the formulas used for Z_{0e} and Z_{0o} when the maximum fractional changes in the circuit characteristics are known. For a coupler, the important circuit characteristics are coupling, directivity, VSWR, and bandwidth.

A. Sensitivity of Coupling Coefficient

The coupling coefficient for a coupled line is defined by [11] as

$$C = \left[\frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}\right]. \tag{2}$$

For a coupled-line directional coupler we can express Z_{0e} and Z_{0o} as

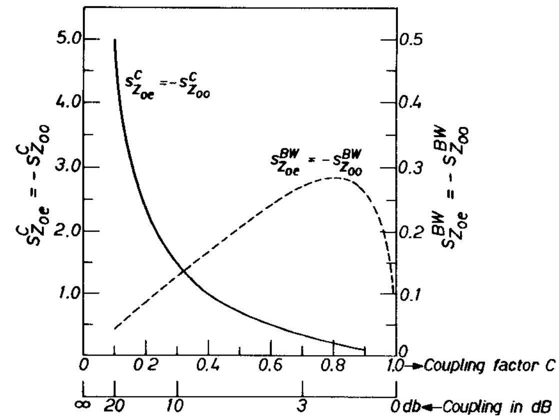

Fig. 1. Sensitivities of coupling and bandwidth with Z_{0e}, Z_{0o} for coupled TEM-line couplers.

$$Z_{0e} = Z_0\left(\frac{1+C}{1-C}\right)^{1/2} \qquad Z_{0o} = Z_0\left(\frac{1-C}{1+C}\right)^{1/2} \tag{3}$$

where

$$Z_0 = (Z_{0e}Z_{0o})^{1/2}. \tag{4}$$

Sensitivities of coupling with respect to Z_{0e} and Z_{0o} can be easily computed from the above relations and written as

$$S^C_{Z_{0e}} = -S^C_{Z_{0o}} = \frac{1-C^2}{2C} \tag{5}$$

These sensitivities have been plotted as a function of coupling in Fig. 1 (continuous curve with the scale on the left-hand side). It is observed that these sensitivities increase for lower values of coupling and become negligibly small when the coupling coefficient approaches unity (zero-dB couplers).

The fractional change in the coupling of a coupler can be found from (1). Since C is a function of two impedances Z_{0e} and Z_{0o}, the maximum fractional change in C is given by

$$\frac{(\Delta C)_{\max}}{C} = \left|\frac{\Delta Z_{0e}}{Z_{0e}} S^C_{Z_{0e}}\right| + \left|\frac{\Delta Z_{0o}}{Z_{0o}} S^C_{Z_{0o}}\right|. \tag{6}$$

The resulting variation of $(\Delta C)_{\max}/C$ with the coefficient of coupling is shown in Fig. 2 (continuous curves with the scale on the left-hand side). For specified tolerances in Z_{0e} and Z_{0o}, $(\Delta C)_{\max}/C$ decreases with increasing values of C. For a change of 2 percent in Z_{0e} and Z_{0o}, the maximum change in coupling is 1.8 dB for a 20-dB coupler, 0.5 dB for a 10-dB coupler, and only 0.2 dB for a 3-dB coupler.

B. VSWR of the Coupler

Sensitivities of Z_0 with respect to Z_{0e} and Z_{0o} are found from (1) and (4),

$$S^{Z_0}_{Z_{0e}} = S^{Z_0}_{Z_{0o}} = 0.5. \tag{7}$$

The maximum fractional change in Z_0 is given by

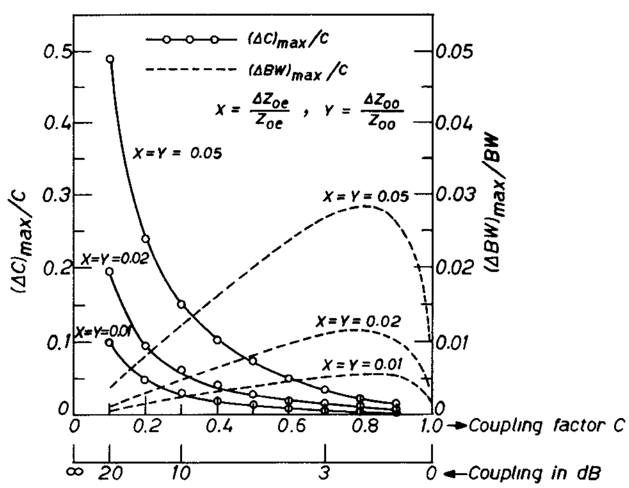

Fig. 2. Fractional change in coupling and bandwidth of coupled TEM-line couplers caused by tolerances in Z_{0e} and Z_{0o}.

$$\frac{(\Delta Z_0)_{\max}}{Z_0} = \left|\frac{\Delta Z_{0e}}{Z_{0e}} S_{Z_{0e}}^{Z_0}\right| + \left|\frac{\Delta Z_{0o}}{Z_{0o}} S_{Z_{0o}}^{Z_0}\right|. \tag{8}$$

The VSWR at the four ports of the coupler is given by

$$\text{VSWR} = \frac{1}{1 - 0.5\left(\left|\frac{\Delta Z_{0e}}{Z_{0e}}\right| + \left|\frac{\Delta Z_{0o}}{Z_{0o}}\right|\right)}. \tag{9}$$

Since the sensitivities in (7) are constant, the VSWR is independent of the coupling coefficient. For fractional deviations in Z_{0e}, Z_{0o} equal to 0.05, 0.02, and 0.01, the values of the VSWR are 1.053, 1.021, and 1.011, respectively.

C. Sensitivity of Bandwidth

The bandwidth of a directional coupler is usually defined in terms of 1-dB points (where the coupled power is 1 dB lower). The ratio of power available at the coupled port to the power incident at the input port (R^2) is given in [11] as

$$R^2 = \frac{C^2 \sin^2\theta}{1 - C^2\cos^2\theta} \tag{10}$$

where θ is the electrical length of the coupled lines. The 1-dB points are obtained from (10) as

$$\theta_{1\,\text{dB}} = \sin^{-1}\left[\left(\frac{m - mC^2}{1 - mC^2}\right)^{1/2}\right] \tag{11}$$

where $m = 0.795$.

If the definition of the bandwidth is modified, the value of m will be different. The fractional bandwidth of the coupler may be written as

$$BW = \frac{4}{\pi}\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{m - mC^2}{1 - mC^2}\right)^{1/2}\right]. \tag{12}$$

From (1) and (12) the sensitivities are found to be

$$S_{Z_{0e}}^{BW} = -S_{Z_{0o}}^{BW} = \left(\frac{C}{BW}\right)\left(\frac{1 + mC}{1 - mC^2}\right)\left[m(1-m)(1-C^2)\right]^{1/2}. \tag{13}$$

These sensitivities are also plotted in Fig. 1 (dotted curve and scale on right-hand side) as a function of coupling. It is observed that sensitivities increase for values of C up to 0.8 where these are maximum (≈ 0.28) and then decrease towards zero as C approaches unity. It may be noted that the bandwidth increases with C as in (11), (12) and is maximum for C equal to unity.

The maximum fractional change in the bandwidth is given by

$$\frac{(\Delta BW)_{\max}}{BW} = \left|\frac{\Delta Z_{0e}}{Z_{0e}} S_{Z_{0e}}^{BW}\right| + \left|\frac{\Delta Z_{0o}}{Z_{0o}} S_{Z_{0o}}^{BW}\right|. \tag{14}$$

Results based on (14) are plotted in Fig. 2 (dotted curves and the scale on right-hand side). The variation with the coupling coefficient is of the same type as the variation of the sensitivity of the bandwidth with the coupling given in Fig. 1. The value of the fractional change in the bandwidth is, however, much smaller (about 1 percent for a 2-percent deviation in impedances).

D. Effects of Impedance Inaccuracies on Directivity

When there are inaccuracies in Z_{0e} and Z_{0o}, we can express Z_0 as

$$Z_0 = (Z_{0e}Z_{0o})^{1/2} = q(Z_{0e}'Z_{0o}')^{1/2} \tag{15}$$

where Z_{0e} and Z_{0o} are the ideal values, and Z_{0e}' and Z_{0o}' are the actual values. In the ideal case ($q = 1$) the directivity is infinite. For other cases ($q \neq 1$) the directivity D is found to be [18]

$$D = \frac{j}{\left(q - \frac{1}{q}\right)}\left[\left(\frac{1+C}{1-C}\right)^{1/2} + \left(\frac{1-C}{1+C}\right)^{1/2}\right]. \tag{16}$$

Values of directivity resulting from tolerances in Z_{0e} and Z_{0o} can be calculated from this relation. For a deviation of 2 percent in Z_{0e} and Z_{0o}, the directivity values are 34 dB, 35 dB, and 37 dB corresponding to coupling values of 20 dB, 10 dB, and 3 dB, respectively.

E. Discussion

We consider an example where $\Delta Z_{0e}/Z_{0e} = \Delta Z_{0o}/Z_{0o} = 0.02$. Then, in the range $0.3 < C < 1.0$ (10 dB–0 dB) there is the worst case deviation of $\Delta C/C = 0.07$ (± 0.5 dB) occuring at $C = 0.3$ (10 dB). In this range of C, the maximum deviations in coupler performance are $(\Delta C)_{\max}/C = 0.07$ and VSWR $= 1.023$ where the directivity $= 34$ dB and $(\Delta BW)_{\max}/BW = 0.011$. These deviations are acceptable for many purposes. However, an inspection of Fig. 2 reveals that for values of C less than 0.3, the accuracy requirements on the formulas is more stringent if we want to keep $(\Delta C)_{\max}/C$ small.

It may be pointed out that sensitivities and fractional changes calculated in this section are valid for the coupled-line directional coupler using any TEM-line configuration. In the following section new closed-form expressions for Z_{0e} and Z_{0o} for coupler microstrips are proposed for coupling coefficients greater than 0.3 (i.e., for coupling tighter than 10 dB). These are used later for studying characteristics of microstrip directional couplers in Section IV.

III. Closed-Form Expressions for Coupled Lines

These expressions are obtained from empirical modifications of an analysis for coupled lines based on an equivalent four-wire structure.

A. Coupled Microstrip Analysis

The even- and the odd-mode impedances of a coupled microstripline, as shown in Fig. 3(a) are obtained by analyzing the four-wire line structure shown in Fig. 3(b) and by using the equivalence between strip and cylindrical structures [12].

For the even mode, equal voltages are applied to the lines 1 and 2 shown in Fig. 3(b). The currents and charges on the four conductors are related by

$$I_1(x)=I_2(x)=-I_3(x)=-I_4(x) \tag{17}$$

and

$$Q_1(x)=Q_2(x)=-Q_3(x)=-Q_4(x). \tag{18}$$

By assuming the spacing between the wires to be large compared to the radius of the wire a, the inductance of this structure is calculated as [13]

$$L=\frac{\mu}{2\pi}\ln\left[\frac{\sqrt{1+(S'/h')^2}}{(a/h')(S'/h')}\right] \tag{19}$$

where $S'>a$, and $h'>a$. For air medium the even-mode impedance is calculated from (19) to be [13]

$$Z_{0e}=60\ln\left[\frac{\sqrt{1+(S'/h')^2}}{(a/h')(S'/h')}\right]\Omega. \tag{20}$$

For the odd-mode impedance, equal and opposite voltages are applied to lines 1 and 2 as shown in Fig. 3(b). The equivalent coupled microstrip structure is similar to those shown in Fig. 3(a) with the polarities on conductors 2 and 3 reversed. The charges and currents in the four wires are now related by

$$I_1(x)=-I_2(x)=I_3(x)=-I_4(x) \tag{21a}$$

$$Q_1(x)=-Q_2(x)=Q_3(x)=-Q_4(x). \tag{21b}$$

The odd-mode impedance for air medium is given by

$$Z_{0o}=60\ln\left[\frac{(S'/h')}{(a/h')\sqrt{1+(S'/h')^2}}\right]\Omega. \tag{22}$$

For the equivalent coupled microstrip Z_{0e} and Z_{0o} are obtained from (20) and (22) by using the following equivalences [12]:

$$a=0.268W+0.335t \quad S'=S+W \quad h'=2h. \tag{23}$$

B. Empirical Modification

The above results hold good for air medium microstrips only. For dielectric substrates the above expressions are modified empirically. For $t/h=0$, the impedances Z_{0e} and Z_{0o} may be written as

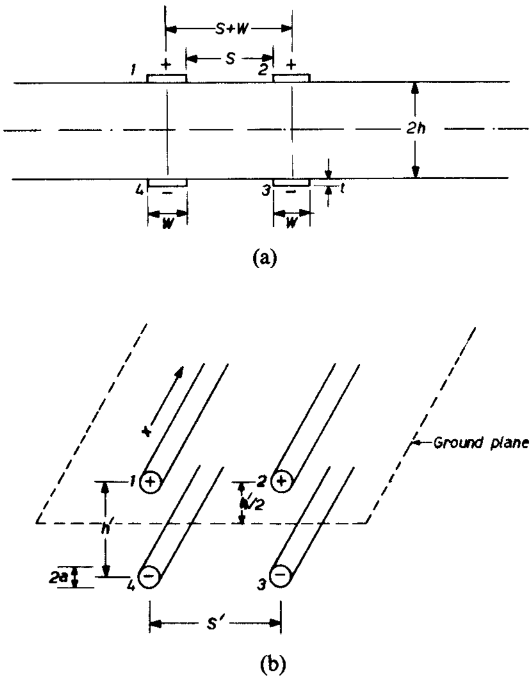

Fig. 3. (a) Coupled microstrip configuration. (b) Equivalent four-wire line in even mode.

$$Z_{0e}=\frac{60}{K_e}\ln\left[\frac{\sqrt{1+(S'/h')^2}}{(a/h')(S'/h')}\right]\Omega \tag{24}$$

$$Z_{0o}=\frac{60}{K_0}\ln\left[\frac{(S'/h')}{(a/h')\sqrt{1+(S'/h')^2}}\right]\Omega \tag{25}$$

where K_e and K_o factors for Z_{0e} and Z_{0o}, respectively, are given by

$$K_{e,o}=\left[\left(\frac{\epsilon_r+1}{2}\right)+\frac{(\epsilon_r-1)}{2\left[1+\frac{10h}{W}\right]^{1/2}\left[1+\frac{v_i(S/h)^2}{(S/h)^3+W/h}\right]}\right]^{1/2} \tag{26}$$

where the value of v_i is 5 for K_e and 10 for K_o.

It is found that relations (24) and (25) combined with (23) have an accuracy of about 9 percent in the following range of circuit parameters:

$$0.2\leqslant S/h\leqslant 2 \quad 0.2\leqslant W/h\leqslant 2 \quad 2\leqslant\epsilon_r\leqslant 80. \tag{27}$$

In order to make these formulas more useful over the practical range of parameters ϵ_r, W/h, and S/h, it is necessary to modify (24) and (25) by incorporating additional empirical factors. With the introduction of these empirical factors, (24) and (25) become, where $t/h=0$,

$$Z_{0e}=\frac{60}{K_eM_e}\ln\left[\frac{\sqrt{1+0.25Q^2}}{(0.067W/h)Q}\right] \tag{28}$$

$$Z_{0o}=\frac{60}{K_oM_o}\ln\left[\frac{Q}{(0.268W/h)\sqrt{1+0.25Q^2}}\right]. \tag{29}$$

The factors M_e, M_o, Q are functions of ϵ_r, W/h, S/h and are listed in the Appendix.

C. Results and Discussion

The values of Z_{0e} and Z_{0o} given by (28) and (29), respectively, show a very good agreement with the results of Bryant and Weiss [8]. In the range

$$0.05 \leq S/h \leq 2 \quad 0.2 \leq W/h \leq 2 \quad 1 \leq \epsilon_r \leq 80 \tag{30}$$

the formulas for Z_{0e} and Z_{0o} show an accuracy of ± 2 percent and ± 2.5 percent, respectively. The relations may hold even for $W/h < 0.2$ and $S/h < 0.05$, but this could not be verified for want of accurate results in this range. Thus (28) and (29) can be used to generate the impedance data for coupled lines subject to the restrictions imposed by (30).

It may be pointed out that the values of Z_{0e} and Z_{0o} obtained by previously available Schwarzmann's formulas [3] differ from the results of Bryant and Weiss [8] by about 20 percent in the range of parameters specified by (30).

IV. Factors Affecting Directional Coupler Performance

Sensitivity analysis of coupled lines is used for finding the fractional changes in impedances caused by the fractional changes in parameters. The changes in parameter values are in turn caused by manufacturing tolerances. The fractional changes in impedance values are used to calculate the values of coupler parameters as discussed in Section II. Closed-form expressions (28) and (29) are needed in the evaluation of these sensitivities.

A. Sensitivities of Z_{0e} and Z_{0o} with Respect to Parameters

From the definition of sensitivity given by (1), the fractional changes in the impedances Z_{0e} and Z_{0o} are related to the tolerances ΔB_n of the independent variables B_n as

$$\frac{(\Delta Z_{0i})_{\max}}{Z_{0i}} = \sum_{n=1}^{4} \left| \frac{\Delta B_n}{B_n} S_{B_n}^{Z_{0i}} \right| \tag{31}$$

where $i = e$ or 0 and B_n's are ϵ_r, h, W, and S.

Equation (31) provides the fractional changes in Z_{0e} and Z_{0o} that can be obtained in practice (limited by manufacturing tolerances). The fractional changes in the values of directional coupler parameters are calculated by

$$\frac{(\Delta R_p)_{\max}}{R_p} = \sum_{n=1}^{4} \left| \frac{\Delta B_n}{B_n} S_{B_n}^{R_p} \right| \tag{32}$$

where R_p's are C, Z_0, and BW.

Sensitivity functions involve derivatives of Z_{0e}, Z_{0o} with respect to ϵ_r, W, h, and S. These are evaluated using the closed-form expressions given in Section III. Expressions for sensitivity functions are lengthy and are not included in the text. These are used in evaluating the fractional changes in coupler parameters using (32). Calculations have been carried out for coupled microstrips on 25-ml thick alumina substrates ($\epsilon_r = 9.6$) when the manufacturing tolerances are $\Delta\epsilon_r = 0.25$, $\Delta W = 0.0001$ in, $\Delta S = 0.0001$ in, and $\Delta h = 0.001$ in. The results are plotted in Figs. 4 and 5. It is noted from these figures that the maximum fractional changes in coupling and bandwidth increase with the increase in the coupling coefficient. It may be pointed out that in Fig. 2 $(\Delta C)_{\max}/C$ decrease with coupling. This plot is for the constant value of the fractional change in impedances and holds good for any TEM-line directional coupler. For $(\Delta C)_{\max}/C$, for microstrip couplers shown in Fig. 4, dimensional tolerances are considered to be constant for various values of C. Values of VSWR shown in Fig. 5 decrease with the increased coupling coefficient. Directivity is seen to improve when the coupling is made tighter. This is in agreement with observation by Levy [18] where the value of q had been taken to be constant.

Contribution of the individual tolerances in ϵ_r, h, W,

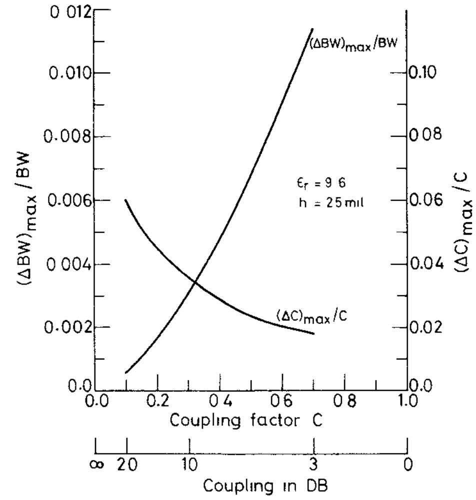

Fig. 4. Fractional change in coupling and coupling bandwidth caused by manufacturing tolerances in ϵ_r, W, h, and S.

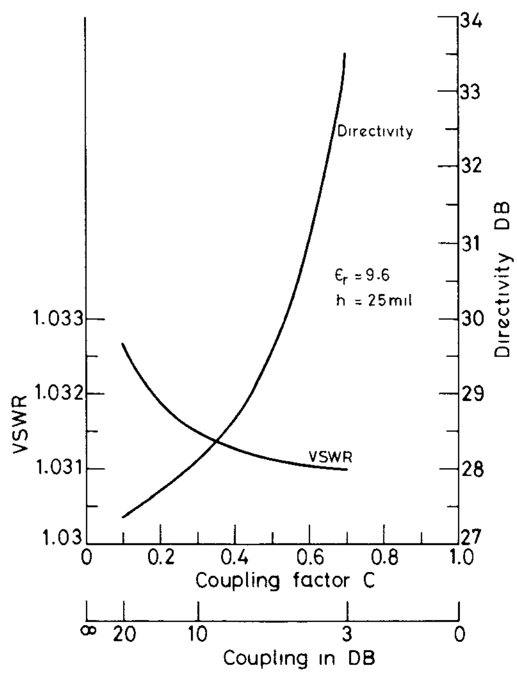

Fig. 5. VSWR and directivity caused by manufacturing tolerances in ϵ_r, h, W, and S.

TABLE I
CONTRIBUTION OF INDIVIDUAL TOLERANCES TO THE CHANGE IN COUPLER PERFORMANCE

	C	Total change (percentage)	Contribution of $\Delta\epsilon_r$	Δh	ΔW	ΔS
$\Delta C/C$	0.1	6.13	0.08	5.6	0.08	0.37
	0.5	2.54	0.02	1.39	0.12	1.01
$\Delta BW/BW$	0.1	0.054	0.0007	0.049	0.0007	0.003
	0.5	0.7	0.01	0.376	0.03	0.28

TABLE II
EFFECTS OF UNEQUAL PHASE VELOCITIES AND DISPERSION ON COUPLER PERFORMANCE: $\epsilon_r=9.6$, $C=0.514$

Parameters	Ideal coupler $V_e = V_o$ all dimensions perfect dB	Coupler with $V_e = V_o$ but with dimensional tolerances dB	Coupler with $V_e \neq V_o$ $\frac{\Delta V}{V}$ = 10.3 percent	Coupler ($V_e \neq V_o$) with dispersion f = 12 GHz
Coupling in dB	-5.78	-5.78 ± 0.22	-5.81	-5.834
$(\frac{\Delta C}{C})$x100	0.0	2.4 percent	0.39 percent	0.6 percent
1 dB coupling bandwidth BW	$\frac{2}{\pi}[1.0638]$	$\frac{2}{\pi}[1.0638 \pm 0.0069]$	$\frac{2}{\pi}[1.0617]$	$\frac{2}{\pi}[1.0227]$
$(\frac{\Delta BW}{BW})$x100	0.0	0.73 percent	0.2 percent	4.1 percent
VSWR	1.00	1.0312	1.0001	1.0027
Directivity	infinite	29.6	18.78	18.31

$\Delta\epsilon_r=0.25$, $\Delta W=0.0001$ in, $\Delta S=0.0001$ in, $\Delta h=0.001$ in.

and S to the percentage changes in coupling and bandwidth are given in Table I. We note that the major contribution to the change in performance is the tolerance in height ($\Delta h=0.001$ in). For tight coupling ($C=0.5$), the tolerance in spacing ($\Delta S=0.0001$ in) has a comparable effect on coupling.

In the range $0.1<C<0.7$ (20 dB–3 dB) the maximum fractional changes in the coupler parameters are $(\Delta C)_{\max}/C=0.61$, VSWR $=1.0327$, $(\Delta \mathrm{BW})_{\max}/\mathrm{BW}=0.0115$, and maximum directivity $=32.00$ dB. It is noted that these fractional changes are permissible for several applications of directional couplers.

B. Effect of Unequal Even- and Odd-Mode Velocities

The changes in coupler performance discussed in Section IV-A have been calculated by assuming equal even- and odd-mode velocities. The effect of unequal even- and odd-mode velocities on coupler parameters is described by Levy [18]. For a typical case ($C=0.514$) calculations for various parameters of the coupler are shown in Table II. Here the results are compared with the effect of the dimensional tolerances. It is observed that the changes in coupling, bandwidth, and the values of VSWR are much less than the changes caused by dimensional tolerances (with $v_e=v_o$). Therefore, the effect of unequal phase velocities can perhaps be ignored in the sensitivity analysis. However, the directivity is mainly governed by unequal phase velocities of even and odd modes.

C. Effect of Dispersion

The analysis in Section IV-B does not take into account the variations of the effective dielectric constant with the frequency. The analysis of a parallel-coupled microstrip, taking dispersion into account, is reported by several authors [14]–[17]. The expression for the frequency dependent effective dielectric constant given in [17] has been used in calculating the coupling, VSWR, directivity, and bandwidth as functions of the frequency. The results for operation at 12 GHz are also included in Table II. It is observed from Table II that for alumina substrates (25 mil thick) changes in coupling and VSWR are negligibly small compared with the changes produced by dimensional tolerances. Unequal phase velocities reduce directivity, and dispersion reduces the bandwidth slightly.

V. APPENDIX

The factors Q, M_e, and M_o appearing in (28) and (29) (Section III-B) are functions of S/h, W/h, and ϵ_r. Denoting S/h by S_1 and W/h by W_1 these factors are given by

$$Q=\left(\frac{S_1+W_1}{2}\right)\left[1+\left(1-\left(\frac{0.536W_1}{S_1+W_1}\right)^2\right)^{1/2}\right]$$

$$M_e=[1-0.18A+0.9B-C-0.9D-E+0.6F-0.05G-0.16H+0.2I-0.125J+0.1K]$$

$$M_o=[1+0.25A'-0.18B'+1.1C'-D'-E'+F'+0.275G'-0.2H'-3I'-6J'-1.5K'+0.15L'+M'-N'-O'+0.2P'-5Q']$$

where the parameters A–K and A'–Q' are listed below:

$$A=\left(\frac{W_1^2}{3+W_1^2}\right)\left(\frac{1}{1+S_1}\right)$$

$$B=\left(\frac{1}{1+W_1}\right)\left(\frac{S_1}{1+S_1}\right)$$

$$C=\frac{0.2}{\epsilon_r^2}\left(\frac{W_1}{1+W_1}\right)\left(\frac{S_1}{1+S_1}\right)$$

$$D=\frac{0.1}{\epsilon_r^2}\left(\frac{1}{1+W_1}\right)\left(\frac{S_1}{1+S_1}\right)$$

$$E=\frac{0.1}{\epsilon_r^2}\left(\frac{W_1}{5+W_1}\right)\left(\frac{1}{1+S_1}\right)$$

$$F=\left(\frac{W_1}{1+W_1+W_1^5}\right)\left(\frac{S_1}{1+S_1+2S_1^2}\right)$$

$$G=\left(\frac{1}{1+2W_1}\right)\left(\frac{S_1}{2+S_1}\right)$$

$$H=\left(\frac{1}{\epsilon_r}\right)\left(\frac{1}{1+0.5W_1+2W_1^3}\right)\left(\frac{S_1}{1+S_1+2S_1^2}\right)$$

$$I=\left(\frac{W_1^2}{3+W_1^5}\right)\left(\frac{S_1^2}{3+S_1^5}\right)$$

$$J=\left(\frac{S_1}{10(S_1)^{1/2}+S_1^2}\right)\left(\frac{1}{1+10W_1}\right)$$

$$K=\left(\frac{W_1^2}{3+W_1^3}\right)\left(\frac{S_1^2}{3+S_1^5}\right)$$

$$A'=\left(\frac{W_1}{1+W_1}\right)\left(\frac{1}{1+10S_1}\right)$$

$$B'=\left(\frac{1}{1+5W_1}\right)\left(\frac{1}{1+5S_1}\right)$$

$$C'=\left(\frac{1}{\epsilon_r}\right)^{1/2}\left(\frac{1}{1+1.5W_1}\right)\left(\frac{1}{3+100S_1}\right)$$

$$D'=\left(1.5+\frac{1}{\epsilon_r^2}\right)\left(\frac{W_1}{1+2W_1}\right)\left(\frac{S_1}{1+10S_1}\right)$$

$$E'=\left(\frac{1}{\epsilon_r}\right)^{1/2}\left(\frac{1}{1+10W_1}\right)\left(\frac{1}{3+100S_1}\right)$$

$$F'=\left(\frac{1}{\epsilon_r^2}\right)\left(\frac{W_1}{1+W_1}\right)\left(\frac{1}{1+100S_1}\right)$$

$$G'=\left(\frac{W_1}{1+W_1+W_1^4}\right)\left(\frac{1}{1+10S_1+100S_1^2}\right)$$

$$H'=\left(\frac{W_1^3}{8+W_1^3}\right)\left(\frac{S_1}{1+S_1+S_1^2+S_1^3}\right)$$

$$I'=\left(\frac{W_1^2}{100+10W_1+W_1^2}\right)\left(\frac{1}{1+100S_1+100S_1^2}\right)$$

$$J'=\left(\frac{1}{1+100W_1+100W_1^2}\right)\left(\frac{1}{1+100S_1+100S_1^2}\right)$$

$$K'=\left(\frac{\epsilon_r}{\epsilon_r+10}\right)\left(\frac{W_1^2}{1+W_1+W_1^2}\right)\left(\frac{S_1^2}{(S_1)^{1/2}+S_1^2+50S_1^3}\right)$$

$$L'=\left(\frac{\epsilon_r}{\epsilon_r+10}\right)\left(\frac{W_1^2}{1+2W_1^3}\right)\left(\frac{S_1^2}{10+S_1^2}\right)$$

$$M'=\left(\frac{0.8}{\epsilon_r^2}\right)\left(\frac{W_1}{2+W_1^2+W_1^3}\right)\left(\frac{S_1^2}{(S_1)^{1/2}+S_1^2+50S_1^3}\right)$$

$$N'=\left(\frac{0.1}{\epsilon_r^2}\right)\left(\frac{W_1^3}{8+W_1^3}\right)\left(\frac{S_1^2}{10+S_1^2}\right)$$

$$O'=\left(\frac{\epsilon_r}{\epsilon_r+10}\right)\left(\frac{W_1}{1+W_1+W_1^2}\right)\left(\frac{S_1^2}{(S_1)^{1/2}+S_1^2+50S_1^3}\right)$$

$$P'=\left(\frac{W_1}{1+W_1+W_1^5}\right)\left(\frac{S_1^2}{2+S_1+S_1^4}\right)$$

$$Q'=\left(\frac{1}{1+100W_1+100W_1^2}\right)\left(\frac{S_1^2}{(S_1)^{1/2}+S_1^2+50S_1^3}\right).$$

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